

Voids in Modified Gravity Reloaded: Eulerian Void Assignment

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ABSTRACT

We revisit the excursion set approach to calculate void abundances in chameleon-type modified gravity theories, which was previously studied by Clampitt, Cai & Li (2013). We focus on properly accounting for the void-in-cloud effect, i.e., the growth of those voids sitting in over-dense regions may be restricted by the evolution of their surroundings. This effect may change the distribution function of voids hence affect predictions on the differences between modified gravity and GR. We show that the thin-shell approximation usually used to calculate the fifth force is qualitatively good but quantitatively inaccurate. Therefore, it is necessary to numerically solve the fifth force in both over-dense and under-dense regions. We then generalise the Eulerian void assignment method of Paranjape, Lam & Sheth (2012b) to our modified gravity model. We implement this method in our Monte Carlo simulations and compare its results with the original Lagrangian methods. We find that the abundances of small voids are significantly reduced in both modified gravity and GR due to the restriction of environments. However, the change in void abundances for the range of void radii of interest for both models is similar. Therefore, the difference between models remains similar to the results from the Lagrangian method, especially if correlated steps of the random walks are used. As Clampitt, Cai & Li (2013), we find that the void abundance is much more sensitive to modified gravity than halo abundances. Our method can then be a faster alternative to N-body simulations for studying the qualitative behaviour of a broad class of theories. We also discuss the limitations and other practical issues associated with its applications.

Key words: large-scale structure of Universe, cosmic voids, modified gravity

1 INTRODUCTION

The discovery of the accelerated expansion of the Universe sparked a surge of research on the possibility of modified gravity models (see, for example, Jain & Khoury 2010; Clifton et al. 2012, for reviews). The main goal of such modifications is to alter the large scale behaviour to explain the acceleration – however, any modifications in the gravity model must at the same time satisfy the tight constraints from the solar system tests. One way to fulfil this requirement is to include some kind of screening mechanism to suppress the modification in the local environment (i.e., regimes with high matter density or deep Newtonian potential). In this work we focus on one particular subclass of modified gravity theories – one that modifies gravity by introducing a dynamical scalar field that mediates a fifth force.

The chameleon model of Khoury & Weltman (2004) is a very representative example of this class of modified gravity theories.

In this model, the acceleration of the Universe is associated with a scalar field, which has a runaway type self-interaction potential and an interaction (coupling) with matter. Such a specific setup ensures that during the cosmic evolution the scalar field is trapped to the vicinity of 0, so that its potential energy roughly stays as a constant – of order the energy density of the cosmological constant in the Λ CDM paradigm – which means that the expansion history can be very close to that of Λ CDM. The coupling to matter produces a fifth force which can be as strong as standard gravity, so that the model is naively ruled out by solar system tests of gravity. However, because the scalar field is trapped close to 0 (especially in regions with high matter density), the fifth force can be severely suppressed, thereby passing the stringent local constraints. This is the chameleon mechanism (Khoury & Weltman 2004), in which the behaviour of the fifth force depends on the environment: the suppression of the fifth force in the solar system is achieved by reducing its force range to sub-millimetres, while on the cosmological background (at late times) the force range can be of order $\mathcal{O}(1 - 10)$ Mpc, on which scales strong deviations from Λ CDM can be found. Notice that on even larger scales, the effect of the fifth force diminishes, implying that the effect on very large-scale structures is minimal.

Because the background evolution and the linear perturbations

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on large scales can be indistinguishable from the standard Λ CDM cosmology (Hu & Sawicki 2007; Li & Barrow 2007; Li & Zhao 2009), and solar system tests are satisfied by construction, non-linear structure formation (on scales of $10^0 \sim 10^2$ Mpc) is the only regime where effects of such models would possibly be detected. A number of studies (e.g., Oyaizu 2008; Oyaizu et al. 2008; Schmidt et al. 2009; Li & Zhao 2009, 2010; Li & Barrow 2011; Zhao, Li & Koyama 2011; Li et al. 2011) employed N-body numerical simulations to study nonlinear structure formation – however high resolution simulations with cosmological volume are still challenging due to the highly non-linear equation governing the scalar field. As a result, semi-analytic methods are often used to make qualitative predictions and provide insights into the underlying physics for such models.

This work will be along this direction and aim at investigating the effect of the chameleon-type modified gravity on the large-scale structure in the nonlinear regime. We will extend the standard excursion set approach (Bond et al. 1991; Mo & White 1996; Sheth & Tormen 1999) to predict the abundance of structures. The application of this approach to halo abundance and bias have been visited by Li & Efstathiou (2012); Li & Lam (2012); Lam & Li (2012), and therefore in this work we will focus on cosmic voids.

Cosmic voids found in redshift surveys have many applications. Observationally, Hoyle et al. (2012) studied the photometric properties of void galaxies found by Hoyle & Vogeley (2002); Pan et al. (2012). There is also ongoing work toward using voids as cosmological probes. The Alcock-Paczynski test has been proposed (Lavaux & Wandelt 2012) and progress has even been made towards applying it to data (Sutter et al. 2012, 2014b). Stacking of voids for the cosmic microwave background has been used to detect the integrated Sachs Wolfe effect as an alternative to the cross-correlation method (Granett et al. 2008; Cai et al. 2014a,b; Hotchkiss et al. 2014; Ilić et al. 2013; Planck Collaboration et al. 2013); Void-void and void-galaxy clustering is another promising cosmological probe (Hamaus et al. 2014). Recent measurements of the weak lensing signal (Melchior et al. 2013) and density profile of voids (Clampitt & Jain 2014) gives a direct handle on their dark matter content. Void ellipticity has been shown to be sensitive to the dark energy equation of state (Bos et al. 2012). Voids properties have been studied in coupled dark energy model using N-body simulations (Li 2011; Sutter et al. 2014).

The excursion set predictions of void abundances have been done previously by Clampitt, Cai & Li (2013), where they showed that voids, due to the under-density and therefore weaker suppression of the fifth force inside them, would yield a stronger signal of modified gravity compared to halos. This work focuses on applying a modern void-assignment algorithm, Eulerian-void assignment, in the context of the same modified gravity model. Eulerian-void assignment was recently proposed by Paranjape, Lam & Sheth (2012b) as an improvement to the traditional Lagrangian-based assignment by taking into account the effect of the immediate surroundings on the growth of voids.

This paper is organised as follows: In § 2 we briefly review the theoretical model to be considered and summarise its main ingredients. In § 3 we find and validate the numerical solutions to the fifth force, which will be used in § 4 to calculate numerically the evolution of spherical over- and under-densities in the modified gravity model. § 5 presents a detailed description of the different void assignment methods adopted in the literature and in this paper; in particular, we will describe how the Eulerian method of Paranjape, Lam & Sheth (2012b) can be implemented in the context of modified gravity. In § 6 we show the comparison of predic-

tions on the void abundance from the different methods and demonstrate their consistency. Finally, § 7 summarises the main results and discusses their implications.

2 THE CHAMELEON THEORY

This section presents the theoretical framework for investigating the cosmological effects of a coupled scalar field. We will give the relevant general field equations in § 2.1, and then specify the models analysed in this paper in § 2.2.

2.1 Cosmology with a coupled scalar field

The equations governing the scalar field can be found in Li & Zhao (2009, 2010); Li & Barrow (2011), and are presented here only to make this paper self-contained. Because of this, this section is brief and contains only the most essential ingredients of coupled scalar field cosmology. Interested readers are referred to the above references for more details.

We start from a Lagrangian density

$$\mathcal{L} = \frac{1}{2} [M_{\text{Pl}}^2 R - \nabla^a \phi \nabla_a \phi] + V(\phi) - C(\phi)(\mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{S}}), \quad (1)$$

in which R is the Ricci scalar, the reduced Planck mass is $M_{\text{Pl}} = 1/\sqrt{8\pi G}$ with G being Newton's constant, and \mathcal{L}_{DM} and \mathcal{L}_{S} are respectively the Lagrangian densities for dark matter and standard model fields. ϕ is the scalar field and $V(\phi)$ is the potential describing its self interaction. The coupling function $C(\phi)$, on the other hand, describes its interaction with matter. The coupled scalar field model is then fully specified by the functional forms for $V(\phi)$ and $C(\phi)$.

Varying the total action with respect to the metric g_{ab} , we obtain the following expression for the total energy momentum tensor in this model:

$$T_{ab} = \nabla_a \phi \nabla_b \phi - g_{ab} \left[\frac{1}{2} \nabla^c \nabla_c \phi - V(\phi) \right] + C(\phi)(T_{ab}^{\text{DM}} + T_{ab}^{\text{S}}),$$

where T_{ab}^{DM} and T_{ab}^{S} are the energy momentum tensors for (uncoupled) dark matter and standard model fields. The existence of the scalar field and its coupling change the form of the energy momentum tensor, leading to potential changes in the background cosmology and structure formation.

The coupling to the scalar field generates an extra interaction (the fifth force) between matter particles, which can be regarded as a result of the exchange of scalar quanta. This is best illustrated by the geodesic equation for dark matter particles

$$\frac{d^2 \mathbf{r}}{dt^2} = -\vec{\nabla} \Phi - \frac{C_\phi(\phi)}{C(\phi)} \vec{\nabla} \phi, \quad (2)$$

where \mathbf{r} is the particle position, t the physical time, Φ the Newtonian potential and $\vec{\nabla}$ is the spatial derivative; $C_\phi \equiv dC/d\phi$. The second term on the right hand side is the fifth force, whose potential (the conservative force potential, not to be confused with the self-interacting potential $V(\phi)$) can be described by $\ln C(\phi)$.

Equation (2) suggest that to follow the motion of particles we need to know the time evolution and spatial configuration of ϕ . This is usually achieved by explicitly solving the scalar field equation of motion

$$\nabla^a \nabla_a \phi + \frac{dV(\phi)}{d\phi} + \rho \frac{dC(\phi)}{d\phi} = 0, \quad (3)$$

where $\rho = \rho_{\text{DM}} + \rho_b$ is the sum of dark matter and baryonic matter densities. Equivalently, the equation of motion can be written as

$$\nabla^a \nabla_a \phi + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0, \quad (4)$$

where we have defined an effective potential for the scalar field

$$V_{\text{eff}}(\phi) = V(\phi) + \rho C(\phi). \quad (5)$$

The background evolution of ϕ can be solved given the present-day value of $\bar{\rho}$ (note that $\bar{\rho} \propto a^{-3}$ (Li & Zhao 2009)). We can then split ϕ into two parts, $\phi = \bar{\phi} + \delta\phi$, where $\bar{\phi}$ is the background value of ϕ and $\delta\phi$ is its perturbation around $\bar{\phi}$, and subtract the background part of the scalar field equation of motion from the full equation to find the equation of motion for $\delta\phi$. In the quasi-static limit, where we can neglect all time derivatives of $\delta\phi$ compared with its spatial derivatives (this is a good approximation on scales well within the horizon), this can be obtained as

$$\bar{\nabla}^2 \delta\phi = \frac{dC(\phi)}{d\phi} \rho - \frac{dC(\bar{\phi})}{d\phi} \bar{\rho} + \frac{dV(\phi)}{d\phi} - \frac{dV(\bar{\phi})}{d\phi}. \quad (6)$$

The calculation of the scalar field ϕ using the above equation then completes the source term of the Poisson equation,

$$\begin{aligned} \bar{\nabla}^2 \Phi &= \frac{1}{2M_{\text{Pl}}^2} [\rho_{\text{tot}} + 3p_{\text{tot}}] - \frac{1}{2M_{\text{Pl}}^2} [\bar{\rho}_{\text{tot}} + 3\bar{p}_{\text{tot}}] \\ &= \frac{1}{2M_{\text{Pl}}^2} [\rho C(\phi) - \bar{\rho} C(\bar{\phi}) - 2V(\phi) + 2V(\bar{\phi})], \end{aligned} \quad (7)$$

where we have neglected the kinetic energy of the scalar field since the running of ϕ is always negligible for the model studied here. In Eq. (7), we have again exploited the quasi-static approximation by dropping time derivatives of the Newtonian potential.

2.2 Specification of Model

As mentioned above, to completely specify the coupled scalar field model, we need to give the functional forms of $V(\phi)$ and $C(\phi)$. Here, for illustration purpose, we will use the chameleon-type models investigated by Li & Zhao (2009, 2010), with

$$C(\phi) = \exp(\gamma\phi/M_{\text{Pl}}), \quad (8)$$

and

$$V(\phi) = \frac{\rho_\Lambda}{[1 - \exp(-\phi/M_{\text{Pl}})]^\alpha}. \quad (9)$$

In the above ρ_Λ is the energy density of the cosmological constant in the standard Λ CDM scenario, ϕ plays the role of dark energy in this model, and γ, α are dimensionless model parameters which control the strength of the coupling $C(\phi)$ and the slope of the scalar field self-interaction potential $V(\phi)$ respectively.

We choose $\alpha \ll 1$ and $\gamma > 0$ as in Li & Zhao (2009, 2010), so that the global minimum of $V_{\text{eff}}(\phi)$ is always very close to $\phi = 0$ throughout the cosmic evolution, and that $m_\phi^2 \equiv d^2 V_{\text{eff}}(\phi)/d\phi^2$ at this minimum is very large in high-density regimes. For example, the minimum of $V_{\text{eff}}(\phi)$ on a background with matter density ρ is approximately (Li & Zhao 2009)

$$\phi = \frac{\alpha\rho_\Lambda}{\gamma\rho}, \quad (10)$$

such that $\phi \ll 1$, especially in high density regions where $\rho \gg \rho_\Lambda$. Such choices of model parameters ensure that:

- ϕ is trapped in the vicinity of $\phi = 0$ throughout the cosmic history and thus $V(\phi) \approx \rho_\Lambda$ behaves as a cosmological constant. For this reason, we take the background expansion of this model to

be exactly the same as that of Λ CDM with the same cosmological parameters. Note that this is not guaranteed if $\alpha \ll 1$ does not hold.

- the fifth force is strongly suppressed in high density regions where ϕ acquires a large mass, $m_\phi^2 \gg H^2$ (H is the Hubble expansion rate), and therefore the fifth force cannot propagate a long distance without decaying. This is because the fifth force, mediated by a scalar field, takes the Yukawa form and decays exponentially over the Compton wavelength, $\lambda \equiv m_\phi^{-1}$, of this scalar field.

The fact that the fifth force is strongly suppressed when matter density is high implies that its influence on structure formation occurs mainly at late times. The environment-dependent behaviour of the fifth force was first considered in Khoury & Weltman (2004), and has since then been known as ‘chameleon screening’. It is one of the most well-studied modified gravity theories: because of the finite range of the fifth force, and because of the severe suppression of it in high-density regions, it is believed that the strongest cosmological constraints on such models come from the study of cosmic voids, which are low density regions ($\delta \sim -0.8$) in the Universe with sizes ranging from a few to $\mathcal{O}(100)$ Mpc.

3 FORCE SOLUTIONS

Although in this work we are only concerned with void abundances, the Eulerian void assignment method of Paranjape, Lam & Sheth (2012b) requires solutions for collapsing over-dense walls around the voids. Thus, the equations below will apply generally to calculate fifth forces for under- and over-densities. The distinction between the two regimes is made entirely in the choice of parameters, specifically the ratio of the object’s density to that of its environment. After presenting the general equations, we will focus on the over-dense solutions since they are unique to this work (under-dense solutions were already shown in detail in Clampitt, Cai & Li (2013)). Finally, we will describe numerical checks of our algorithm.

3.1 Scalar field solution

We are interested in the simplest model of a dark matter halo (or void), with top-hat (or bucket) density profiles described by

$$\rho_0(\chi) = \begin{cases} \rho_{\text{in}} & \text{for } \chi \leq r \\ \rho_{\text{out}} & \text{for } \chi > r \end{cases}, \quad (11)$$

in which r is the object’s radius and χ is a variable characterising the distance from the centre of the object. Because ϕ is confined to the vicinity of $\phi = 0$, we have $C(\phi) \approx 1$ and the first term on the right-hand side of Eq. (7) can be integrated once to find the force per unit test mass

$$F_N(\chi) = -\frac{GM(<\chi)}{\chi^2}. \quad (12)$$

The second term on the right-hand side of Eq. (2) is the fifth force, which for our choice of $C(\phi)$ in Eq. (8) can be expressed as

$$F_\phi(\chi) = -\gamma \frac{d}{d\chi} (\phi/M_{\text{Pl}}). \quad (13)$$

We define the ratio of fifth to Newtonian forces at the surface of the object ($\chi = r$) as

$$\eta \equiv \frac{F_\phi}{F_N} = \frac{6\gamma M_{\text{Pl}}}{r\rho_{\text{in}}} \left. \frac{d\phi}{d\chi} \right|_{\chi=r}, \quad (14)$$

which is constrained to be $\eta \leq 2\gamma^2$ for over-densities.

In the model considered in this work, the force ratio η can be equivalently determined by three length scales: the radius of the object r , and the Compton wavelengths inside and outside the object. The latter are given by the following relations:

$$\rho_i = \left(\frac{M_{\text{Pl}} \sqrt{\alpha \rho_\Lambda}}{\gamma} \right) \frac{1}{\lambda_i} \quad (15)$$

where $i = [\text{in}, \text{out}]$ and λ is the Compton wavelength of the scalar field (note that the Compton wavelength is small on a high matter-density background and vice versa). In Clampitt, Cai & Li (2013), we showed that the degrees of freedom of a spherical top-hat under-density can be reduced from the three length scales, r , λ_{in} , and λ_{out} , to two ratios of these lengths: r/λ_{out} and $\lambda_{\text{out}}/\lambda_{\text{in}}$. This logic proceeds without change for over-densities, and so we only quote the key equations here.

The effective potential for the specific model studied here can be written as

$$V_{\text{eff}}(\phi) = \frac{\Lambda}{[1 - \exp(-\phi/M_{\text{Pl}})]^\alpha} + \rho_i \exp(\gamma\phi/M_{\text{Pl}}), \quad (16)$$

with which the equation of motion Eq. (4) can be simplified to read

$$\frac{d^2\psi}{d\tau^2} + \frac{2}{\tau} \frac{d\psi}{d\tau} + \frac{1}{\psi} = \begin{cases} \lambda_{\text{out}}/\lambda_{\text{in}} & \text{for } \tau \leq r/\lambda_{\text{out}} \\ 1 & \text{for } \tau > r/\lambda_{\text{out}} \end{cases} \quad (17)$$

with the following two boundary conditions

$$\left. \frac{d\psi}{d\tau} \right|_{\tau=0} = 0, \quad \psi(\tau \rightarrow \infty) = 1. \quad (18)$$

Note that in the above the solution has been rescaled and written in terms of $\psi \equiv \phi/\phi_{\text{out}}$ and $\tau \equiv \chi/\lambda_{\text{out}}$. Rewriting Eq. (14) in terms of the new variables and using $\rho_\Lambda = \Omega_\Lambda \rho_c$ (where ρ_c is the critical density at the present day), we find

$$\eta(r) = 6\gamma \sqrt{\frac{\alpha \Omega_\Lambda}{\Omega_m}} \frac{M_{\text{Pl}} \sqrt{\bar{\rho}_m}}{r \rho_{\text{in}}} \left. \frac{d\psi}{d\tau} \right|_{\tau=r/\lambda_{\text{out}}}, \quad (19)$$

where $\bar{\rho}_m$ is the background matter density today.

The recasting of the scalar field equation of motion in the form of Eq. (17) has the advantage that the fifth-force-to-gravity ratio $\eta(r)$ can then be obtained by interpolating a 2D pre-computed table, for spherical top-hat systems of arbitrary size, density, and in any kind of environment. This can in turn greatly simplify the numerical computations in this work, which would otherwise be prohibitively expensive.

3.2 Comparison to the thin-shell approximation

The radial profile of a chameleon-type scalar field has been studied in detail for spherical over-densities (see, e.g., Khoury & Weltman 2004; Lombriser et al. 2012; Davis et al. 2012, for a few examples). Khoury & Weltman (2004) derived a simple analytical formula for the fifth force which has been shown to agree well with numerical simulations (Li, Zhao & Koyama 2012). Here we show that the thin-shell prescription followed by Li & Efstathiou (2012) (rewritten using our reduced parameters) given by

$$F_\phi^{(\text{thin-shell})}/F_N = \frac{\lambda_{\text{in}} \lambda_{\text{out}}}{r^2} - \left(\frac{\lambda_{\text{in}}}{\lambda_{\text{out}}} \right)^2, \quad (20)$$

gives qualitatively reasonable results.

In Fig. 1 we compare the two solutions, where the solid lines in the top panel show the numerical solutions for the ratio of forces η as a function of the two dimensionless parameters r/λ_{out} and

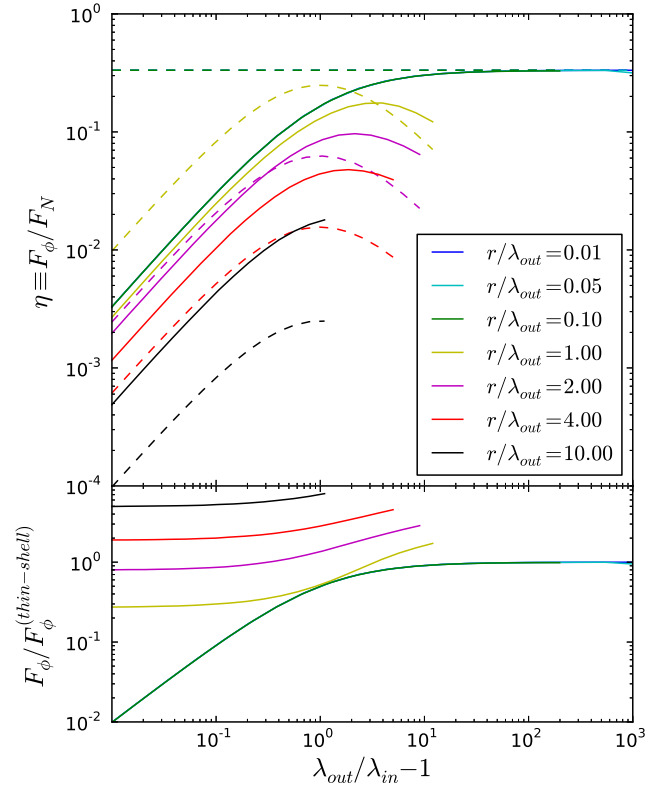


Figure 1. (top panel): We compare the value of the force ratio $\eta = F_\phi/F_N$ in both the exact solution (solid lines) and the thin-shell approximation (dotted lines). For $r/\lambda_{\text{out}} > 10$ the fifth force is less than 2% of the strength of gravity. Thus we do not need the exact solution and can approximate the fifth force as zero there. (bottom panel): The ratio of the curves in the top panel. The approximate solution is accurate for small r/λ_{out} , large $\lambda_{\text{out}}/\lambda_{\text{in}}$ values, but otherwise does not reproduce well the exact result. At small $\lambda_{\text{out}}/\lambda_{\text{in}}$ however, the fifth force is quite weak anyway, so we can set it to zero safely.

$\lambda_{\text{out}}/\lambda_{\text{in}}$, while the dotted lines show the thin-shell approximation. $\lambda_{\text{out}} > \lambda_{\text{in}}$ means that the density inside the top-hat region is greater than outside, which is the case for haloes. Both solutions are qualitatively the same:

- When the exterior and interior Compton wavelengths are very close in magnitude, the fifth force is negligible compared to gravity, because the scalar field takes very similar values inside and outside the over-density, making its gradient (i.e., the fifth force) small.

- As λ_{out} increases, the fifth force grows in importance until $\lambda_{\text{out}} \sim \text{a few } \times \lambda_{\text{in}}$, after which the ratio $\eta = F_\phi/F_N$ decreases. This can be understood as follows: suppose λ_{in} remains unchanged, then an increase in λ_{out} with fixed r/λ_{out} means that the radius of the over-density, and so the Newtonian potential $\Phi_N \propto r^2 \propto \lambda_{\text{out}}^2$, is increased; meanwhile, according to the thin-shell approximation, the fifth-force-to-gravity ratio is proportional to $|\phi_{\text{out}} - \phi_{\text{in}}|/\Phi_N$, with $\phi_{\text{out}} \approx \alpha \rho_\Lambda / \gamma \rho_{\text{out}} \propto \lambda_{\text{out}}$ for our model: when ϕ_{out} is close to ϕ_{in} , i.e., when $\lambda_{\text{out}}/\lambda_{\text{in}} \sim 1$, $|\phi_{\text{out}} - \phi_{\text{in}}|$ increases faster than Φ_N , making η increase; but when with $\phi_{\text{out}} \gg \phi_{\text{in}}$, η becomes $\propto |\phi_{\text{out}}/\Phi_N| \propto \lambda_{\text{out}}^{-1}$, which decreases with λ_{out} . Physically, there are two parts of the screening, self and environment screening: the self screening becomes stronger with increasing mass (and therefore increasing r), while the environment screening becomes

weaker with decreasing environmental density (and therefore increasing λ_{out}). The behaviour of η , with r/λ_{out} fixed, is the result of the competition of the two.

- The fifth force ratio η decreases monotonically with increasing radius r of the spherical over-density, for fixed λ_{out} . This is because a larger r means that the over-density is more massive, and therefore more efficient self-screening of the fifth force in given environment (specified by λ_{out}).

While it is qualitatively very good, the thin-shell solution can be up to a factor of ten too large or too small, as shown by the ratio of exact to thin-shell solutions in the bottom panel of Fig. 1. The thin-shell solution is quantitatively correct only in the regime in which the exterior Compton wavelength is much larger than both the over-density radius and its interior Compton wavelength. This is where the fifth force has its largest magnitude relative to gravity, $F_\phi = F_N/3$, the thick-shell regime, and for other regimes we need to go beyond the thin-shell approximation to be accurate. Furthermore, because the Eulerian environment results of this work require the fifth force solution for both over- and under-densities (and because the thin-shell approximation fails for the latter), we need the exact (numerical) solution in both cases for the sake of continuity and consistency.

3.3 Validation of numerical solution

Having demonstrated that a numerical solution of the fifth force is necessary for both under- and over-densities, we proceed to the non-trivial validation of such solutions. The void solutions of Eq. (17) correspond to the parameter space with $\lambda_{\text{out}}/\lambda_{\text{in}} < 1$. These are shown in Clampitt, Cai & Li (2013) and are numerically straightforward. However, for the over-dense case ($\lambda_{\text{out}}/\lambda_{\text{in}} > 1$), much of the parameter space is difficult to solve. In Fig. 2 we show these solutions of Eq. (17) as a function of the two dimensionless parameters. In the top-right corner of this plot we were unable to obtain numerical solutions, and therefore set $d\psi/d\tau = 0$ for display purposes. However, according to Fig. 1, the fifth force can still be a significant fraction of gravity in this regime (more precisely, it can have a strength $> 2\%$ of gravity.)

Fortuitously, over-densities with the range of Eulerian radii in which we are interested ($R_{\text{eul}} \geq 5\text{Mpc}/h$) do not stray into this unsolved regime. The solutions of Sec. 4 require calculating the evolution of an over-density from deep in the matter dominated era to the present day. Early in cosmic history the r/λ_{out} parameter is very large (because both r is large and λ_{out} is quite small due to the high density) and $\lambda_{\text{out}}/\lambda_{\text{in}}$ is only slightly larger than one. Then, for collapsing over-dense regions $\lambda_{\text{out}}/\lambda_{\text{in}}$ grows monotonically with time, while r/λ_{out} decreases monotonically with time. Qualitatively, then, all solutions track from the top left of the Fig. 2 force table to the bottom right as time passes.

The difficulty of obtaining fifth force solutions is greater for larger objects, corresponding to larger smoothing radii in the initial Lagrangian density field or equivalently smaller s . (Here $s = \sigma^2$ is the variance of the smoothed linear matter power spectrum, to be defined below.) However, we have verified that we can obtain the Eulerian barriers (again to be defined below) all the way to a small value of s , that is, down to some s_{min} such that

$$\delta_{\text{lin}} > 5\sqrt{s_{\text{min}}} = 5\sigma. \quad (21)$$

We do not need the barrier solution at such small s since they are seldom hit by random walks starting at the origin. In summary, we have verified that for each timestep with $F_\phi \geq 0.02F_N$, we solve

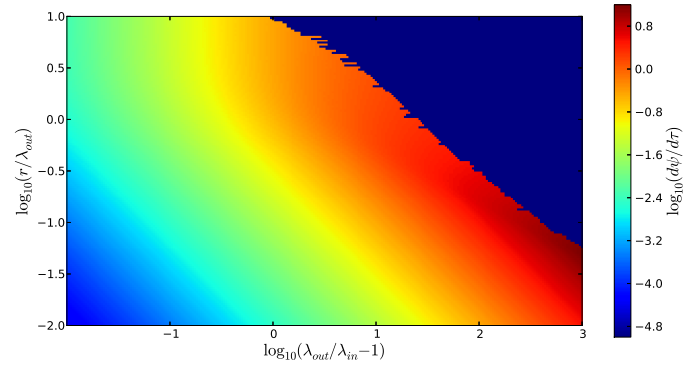


Figure 2. The solution of Eq. (17) as a function of dimensionless parameters r/λ_{out} and $\lambda_{\text{out}}/\lambda_{\text{in}}$, in the over-density case for which $\lambda_{\text{out}}/\lambda_{\text{in}} > 1$. The fifth force $F_\phi \propto d\psi/d\tau$ grows monotonically with either quantity, for all relevant parameter space. The top-right corner is numerically difficult to solve, but we have verified that this regime of parameter space is not used in obtaining the Eulerian barriers.

the exact numerical solution. For timesteps with $F_\phi < 0.02F_N$, we set the fifth force to zero.

4 EVOLUTION OF DENSITY PERTURBATIONS

In our simplified modelling of the evolution of spherical over- and under-densities, we make the assumption that the profile remains as a top-hat or bucket during the whole course and there are no shell crossings. Under this premise, we need to track the evolution of two shells: the halo (for over-density cases) or the void (for under-density cases) shell, with proper radius $r_{\text{in}}(t)$, as well as the shell of the environment that co-evolves with the over- or under-density inside it (Li & Efstathiou 2012), with radius $r_{\text{env}}(t)$. The evolution equation is given by

$$\frac{\ddot{r}_j}{r_j} = -\frac{1}{6M_{\text{Pl}}^2} [\rho_j(1 + \eta) - 2\rho_\Lambda]. \quad (22)$$

where $\rho_j \equiv 3M_j/4\pi r_j^3$ is the matter density in the spherical shell and the constant $\rho_\Lambda \approx V(\phi)$ is the effective dark energy density defined in § 3.1. The notation r_j denotes r_{in} or r_{env} .

The only difference between the evolution of the inner shell and that of its environment is the effect of the fifth force. For the inner shell we calculate the fifth force, whereas the larger environment shell is approximated as evolving under GR, $\eta = 0$, because the environment is generally taken to be larger than the Compton wavelength of the scalar field, for which the fifth-force-to-gravity ratio is small [cf. Fig. 1]. To calculate the fifth force on the inner shell at each time-step we use a spherical top-hat profile,

$$\rho(\chi) = \begin{cases} \rho_{\text{in}} & \text{for } \chi \leq r_{\text{in}} \\ \rho_{\text{env}} & \text{for } \chi > r_{\text{in}} \end{cases}. \quad (23)$$

The fifth-force-to-gravity ratio is then, using Eq. (19),

$$\eta = \frac{\sqrt{3\alpha\Omega_\Lambda}\gamma\frac{d\psi}{d\tau}\big|_{\tau=r/\lambda_{\text{env}}}}{\frac{1}{2}\Omega_m(H_0R)(ay_{\text{in}})^{-2}}, \quad (24)$$

for the inner shell. Following the general excursion set literature, in Li & Efstathiou (2012) and Clampitt, Cai & Li (2013), Eq. (22) is recast to a simpler form by making several changes of variable:

they define $N \equiv \ln(a)$ and $y_j(t) \equiv r_j(t)/a(t)R_j$, where R_j is the initial comoving radius. Derivatives with respect to N are denoted by $y' = dy/dN$. We do not repeat in detail here, but only show the result from those references:

$$y_j'' + \left[2 - \frac{3}{2}\Omega_m(N)\right] y_j' + \frac{\Omega_m(N)}{2} [y_j^{-3}(1 + \eta) - 1] y_j = 0, \quad (25)$$

with initial conditions specified at $a_{\text{ini}} \ll 1$ (deep into the matter dominated epoch), as

$$y_j(a_{\text{ini}}) = 1 - \delta_{j,\text{ini}}/3, \quad y_j'(a_{\text{ini}}) = -\delta_{j,\text{ini}}/3. \quad (26)$$

Note that in the above, $\Omega_m(N) \equiv \Omega_m e^{-3N}/(\Omega_m e^{-3N} + \Omega_\Lambda)$, and $\Omega_\Lambda(N) \equiv \Omega_\Lambda/(\Omega_m e^{-3N} + \Omega_\Lambda)$. $\delta_{j,\text{ini}}$ is the initial density perturbation of the spherical patch ($j = \text{in}$) or its environment ($j = \text{env}$).

Thus, the solution is given by considering two second-order differential equations, Eq. (25) with $j = [\text{in}, \text{env}]$ along with the boundary conditions given in Eq. (26). The force ratio η in Eq. (24) at arbitrary time is obtained by interpolating the table (cf. Fig. 2) for both halos (over-densities, cf. Fig. 2) and voids (underdensities), which is straightforward provided the values of r/λ_{out} and $\lambda_{\text{out}}/\lambda_{\text{in}}$ at that time.

5 VOID ASSIGNMENT IN THE EXCURSION SET: LAGRANGIAN VS. EULERIAN

In this section we first briefly review some of the key aspects of a recently proposed void assignment formalism based on Eulerian arguments (in contrast to the traditional Lagrangian-based approach). We will then describe how to extend this Eulerian-based void assignment to modified gravity theories. For clarity we will choose a single set of parameters $(\alpha, 2\gamma^2) = (10^{-6}, 1/3)$ to demonstrate the formalism, and in a future work we plan to apply the methodology developed in this work to study void abundances in a class of chameleon-like theories.

5.1 Void assignment in excursion set formalism

The excursion set formalism (Bond et al. 1991) was initially applied to describe the abundance of halos: the calculation is mapped to the computation of the first crossing distribution across some prescribed barriers (see, for example, Zentner 2007, for a review). In its simplest form, the first crossing across a constant barrier whose amplitude is obtained from the spherical collapse model is evaluated for random walks whose heights depend on the linearly extrapolated matter power spectrum as well as the smoothing window filter: analytic solutions are available for sharp- k filter (Zhang & Hui 2006; Lam & Sheth 2009). Recent developments improve the model by including scale-dependent (Sheth & Tormen 1999, 2002) barriers, diffusive barriers (Maggiore & Riotto 2010; Corasaniti & Achitouv 2011), random walks with correlated-steps (Maggiore & Riotto 2010; Paranjape, Lam & Sheth 2012a; Musso & Sheth 2012), and peak constraints (Paranjape & Sheth 2012; Paranjape, Sheth & Desjacques 2013).

The formation of voids, the biggest under-dense regions in the Universe, can also be formulated in the excursion set formalism. Sheth & van de Weygaert (2004) demonstrated that this calculation has to include an additional criterion in order to avoid over-counting the number of voids: the so-called void-in-cloud effect in which voids sitting in over-dense regions where halos are forming should be excluded. Technically, this is realised in the excursion

set formalism by imposing two barriers for halo (denoted by δ_c) and void (denoted by δ_v) formation respectively. One then computes the first down-crossing probability across the void formation barrier of random walks that *never* crossed the halo formation barrier at larger smoothing scales. Sheth & van de Weygaert (2004) provided an analytical solution for the case where both halo and void barriers are constant; Lam, Sheth & Desjacques (2009) generalised the solution to barriers of arbitrary shapes. Some more recent attempts of formulating void abundance include Eulerian-void assignment (Paranjape, Lam & Sheth 2012b, more details below), modifying the mapping from Lagrangian volume to abundance (Jennings, Li & Hu 2013), and introducing diffusive barriers (Achitouv, Neyrinck & Paranjape 2013). In this work we will focus on extending the Eulerian-void assignment in a context in which gravity is modified.

The main purpose of the Eulerian-void assignment is to look for the biggest Eulerian volume that satisfies the void criteria: having a density below 20% of the background. Conservation of mass requires the Lagrangian patch to expand 5 times (in volume). Another essential assumption in the formalism is no shell crossing: while the relative separations change, concentric shells (and hence the mass within them) preserve their orderings (see, for example, Sheth 1998, for discussion). Hence the immediate environment would have significant impact on the formation of voids: the void-in-cloud described above is a special case in which the surrounding environment collapses into a vanishing Eulerian size. (Note that it is only a limiting case in which the spherical collapse approximation results in vanishing volume. In reality halos are virialised objects with physical sizes). However, this is only part of the picture: halo-forming is not a necessary condition to modify the formation of voids. For example, if an under-dense region is embedded in a slightly over-dense environment, the comoving volume of this slightly over-dense environment would decrease and restrict the expansion of the under-dense patch. Whether a void would be formed depends on the *Eulerian* size of the surrounding over-dense environment. There are three possibilities:

- A void of the same size as predicted by the Lagrangian treatment (Sheth & van de Weygaert 2004) will be formed;
- A void will be formed, however its size is smaller than its Lagrangian treatment counterpart;
- No void will be formed (in other words the limiting case where the void size is zero).

As a result, the Eulerian-volume void assignment consists of a remapping of the size of the void being formed and the volume function of voids is modified accordingly. For clarity we refer to this Eulerian void assignment as the PLS (Paranjape, Lam & Sheth 2012b) void assignment in the following.

The evolution of the Eulerian volume of the surrounding environment is a key ingredient in the PLS void assignment. In Paranjape, Lam & Sheth (2012b), they use the following spherical collapse approximations (Bernardeau 1994; Sheth 1998):

$$\Delta_{\text{NL}} \equiv \frac{M}{\bar{\rho} V_E} \approx \left(1 - \frac{\delta_l}{\delta_c}\right)^{-\delta_c} \quad (27)$$

to obtain the relationship between the Lagrangian volume (equivalently, the total enclosed mass M) and the linearly extrapolated density contrast ($B_{V_E} = \delta_l$) for a given Eulerian volume V_E :

$$B_{V_E}(M) = \delta_c \left[1 - \left(\frac{M}{\bar{\rho} V_E}\right)^{-1/\delta_c}\right]. \quad (28)$$

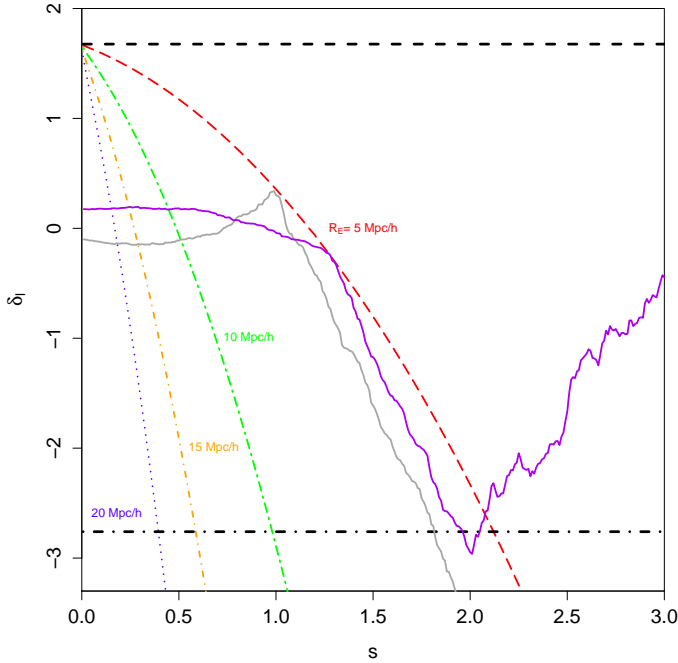


Figure 3. Illustration of Eulerian-void assignment. Barriers for four different Eulerian volumes are shown, together with the constant barriers for halo (top thick dashed) and void (bottom thick dot-dashed) formation. Two sample random walks shown demonstrate the characteristics of Eulerian vs Lagrangian void assignment (see text for descriptions).

$B_{V_E}(M)$ defines a barrier in the $\delta_l - s$ plane for Eulerian volume V_E , where S is the variance of the smoothed linear power spectrum over a Lagrangian volume V_L which satisfies $M = \bar{\rho}V_L$,

$$s \equiv \sigma^2(M) = \int \frac{d^3k}{(2\pi)^3} P(k) W^2(kR_L), \quad (29)$$

in which $W(kR_L)$ is the Fourier transform of the smoothing window function and R_L is the Lagrangian radius associated with the Lagrangian volume V_L .

Note that Eq. (28) defines a monotonically decreasing barrier in the $\delta_l - s$ plane and its limiting value for $\Delta_{NL} \rightarrow \infty$ is δ_c . When $\Delta_{NL} = 0.2$ and $\delta_c = 1.676$, $B_{V_E, \Delta_{NL}=0.2}(M) \approx -2.76$ which is the constant void formation barrier. Sheth (1998); Lam & Sheth (2008a) discussed the application of the excursion set with barrier defined in Eq. (28) to obtain the mass distribution at some fixed Eulerian volume.

Changing V_E in Eq. (28) results in a series of nested curves whose heights are lower for increasing V_E . The limiting case where $V_E \rightarrow 0$ is a constant barrier $B_{V_E} = \delta_c$ and this corresponds to the halo formation barrier. The PLS algorithm looks for the biggest Eulerian volume which has a density contrast of -0.8 : we are looking for random walks that cross some Eulerian barriers below the threshold $\delta_v = -2.76$. For each random walk, we start from a very large Eulerian volume. Since the Eulerian barrier associated with a big volume is decreasing very rapidly, the random walk almost always pierce that barrier above δ_v . As we gradually decrease the Eulerian volume, the flattening of the barrier allows the random walk fluctuates before piercing the barrier. Ultimately one may find an Eulerian volume whose barrier is only pierced by the random walk below δ_v and this Eulerian volume is the Eulerian size of the void associated with the random walk.

Figure 3 uses two sample random walks to illustrate the char-

acteristic of PLS void assignment (note: these walks are for demonstration purposes and while non-vanishing heights at $s = 0$ are unphysical, they do not affect our discussion). Two thick horizontal curves are halo (top) and void (bottom) thresholds in GR. Four decreasing barriers show Eulerian barriers for different Eulerian sizes. Both walks represent void forming regions in Lagrangian-void assignment: they cross the bottom barrier (at approximately $s = 1.8$ and 2 respectively) without crossing the upper constant barrier at smaller s , and as such they represent voids of similar sizes in the Lagrangian void assignment scheme. The result is very different when Eulerian-void assignment is considered: the grey walk is going to form a void of Eulerian radius of $5 \text{ Mpc}/h$ while the purple walk is not a void (in the regions shown). The red decreasing barrier corresponds to an Eulerian size $R_E = 5 \text{ Mpc}/h$. The grey walk touches it at around $s = 1$ but the walk never pierces this barrier above δ_v . To see it is the biggest Eulerian size allowed for the grey random walk, imagine we increase the Eulerian size by ΔR_E . The Eulerian barrier associated with $R_E + \Delta R_E$ lies somewhere between the green and red barriers – the grey random walk will pierce such barrier with $\delta_l > \delta_v$, and as a result the density contrast within $R_E + \Delta R_E$ will be higher than 20% of the background. This sudden increase in mass corresponds to the wall around the void as discussed in Paranjape, Lam & Sheth (2012b).

The purple random walk has a different story: although it crosses the δ_v barrier without crossing the δ_c barrier, the random walk pierces the red Eulerian barrier at around $s = 2.1$, above the δ_v barrier. For this particular walk one would need to consider smaller Eulerian sizes (and hence the barrier will be shallower) to check if a void would be formed (not shown here).

We would like to point out two important characteristics of this PLS void assignment as compared to the Lagrangian version:

- (i) Random walks identified as void-forming in Lagrangian-void assignment (crossing δ_v at some S but never crossed δ_c for $s < S$) may not form a void. However the reverse is true: PLS assigned voids are always associated with a Lagrangian counterpart.
- (ii) PLS assigned voids cannot be bigger than the corresponding Lagrangian-assigned voids.

5.2 Void assignment in modified gravity models

The above discussion on Eulerian-void assignment applies to models where the gravity is described by general relativity. In this section we describe how to extend the various void assignments to modified gravity models. Clampitt, Cai & Li (2013) discussed the extension of Lagrangian-void assignment and found that the abundance of voids would constrain modified gravity models. In particular, the signature of modified gravity is stronger at the bigger voids, which also justifies their approximation of neglecting the void-in-cloud effect. In what follows we review how to apply the Lagrangian void assignment in modified gravity models as well as describe how to apply the PLS void assignment.

In models with modified gravity the dynamics is modified by the presence of the fifth force, whose strength can depend on the environmental density. Thus all the barriers involved are modified accordingly. There are multiple scales involved in this case: an environmental scale corresponding to halo formation, as well as various Eulerian volumes and their corresponding environments against which the void formation criteria is checked. As discussed in Li & Efstathiou (2012), the environment should be big enough to encompass the objects being considered, but at the same time give a representative environment to that particular object (hence it

cannot be too large). Li & Lam (2012); Lam & Li (2012) proposed using an Eulerian size of $R_h = 5 \text{ Mpc}/h$ (which was roughly the Compton wavelength of the scalar field in the models considered) as the environment for halo formation and we will adopt the same here. For void formation, Clampitt, Cai & Li (2013) suggested a scale 5 times the void radius as the environment, each we also adopt here. While Lagrangian environment is used in Clampitt, Cai & Li (2013), we use Eulerian environment in the following since it allows us to follow the evolution of the Eulerian volumes closely. Notice that previous studies always assume the evolution of the environment follows that of GR since it is always much bigger than the Compton wavelength of the scalar field being considered. We will also make this assumption in the following discussion.

We apply the PLS void assignment as follows. We first generate the following tables of different barriers for different environmental densities δ_{env} (here δ_{env} is the linear density contrast of the environment, extrapolated to today using the Λ CDM linear growth factor; $\delta_{\text{env}} > 0$ denotes overdense environments and vice versa):

- (i) Halo formation barrier (in GR it is a constant barrier at δ_c);
- (ii) Void formation barrier (in GR it is again a constant barrier at δ_v);
- (iii) Eulerian barriers $B_V(S|\delta_{\text{env}})$ for each Eulerian volume V_E .

Although we assume that the evolution of all the environments would be described by GR, we use the same numeric solver to obtain the GR Eulerian volume barrier for consistency. Figure 4 shows results for different barriers. For an Eulerian size of $5 \text{ Mpc}/h$, the blue-shaded region corresponds to its Eulerian barriers for a wide range of environmental density contrast ($-2.4 \leq \delta_{\text{env}} \leq 1.5$). The red dashed curve running from the top of the shaded region at small s to the bottom of the shaded region at big s is the GR Eulerian barrier computed by the same numerical solver. On the other hand the green dotted curve shows the approximation formula Eq. (28).

Figure 5 shows the difference between barriers of three different Eulerian sizes from the GR counterparts. Three environmental density contrasts ($\delta_{\text{env}} = -2.4, 1.5, 0$) are chosen for comparison, and the result shows the following features:

- At very large scales (small s), there is no difference between the GR and modified gravity barriers. At larger s , the two deviate from each other: the deviation starts at smaller s for bigger Eulerian size since, with δ_{env} fixed, to have a bigger Eulerian size the initial density δ_i is in general lower (cf. Fig. 4), which means the effect of the fifth force is stronger.
- Whether the Eulerian barriers in modified gravity are higher or lower than their GR counterparts depends on the relative density contrast to the environment. If the density inside is higher than that in the environment, the fifth force is attractive (i.e., points towards the centre) and slows down the expansion. To have the same R_E today the patch needs to have a lower initial density which helps to speed up the expansion. If the density inside is lower than that in the environment, the fifth force helps the expansion, which has to be compensated by having higher initial densities in the patch. A special case is when $\delta_i = \delta_{\text{env}}$, so that the fifth force vanishes identically throughout the evolution: here $\delta_i = \delta_i^{\text{GR}}$, and is where the curves cross zero in Fig. 5. As an example, taking the dotted curves in Fig. 5, which correspond to a very dense environment ($\delta_{\text{env}} = 1.5$) so that the barrier is higher in modified gravity for most values of s ; however, when s is extremely small, the initial size of the patch is huge and for it to evolve to a given R_E today its initial density has to be higher than 1.5, so that the patch must be over-dense. In this case, the modified gravity barrier becomes

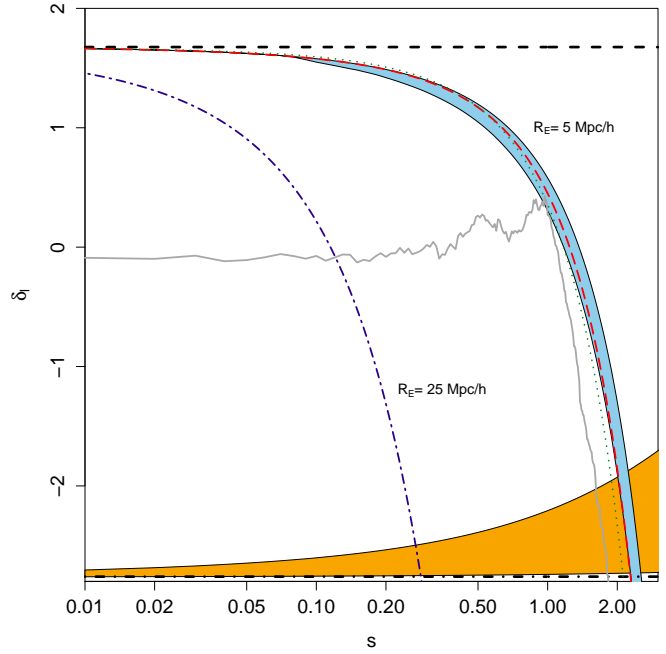


Figure 4. Illustration of the PLS void assignment in modified gravity models (see text for description). The constant barriers at the top and the bottom are the halo and void formation threshold respectively in GR. The orange shaded region is the void formation barriers in modified gravity model for $-2.4 \leq \delta_{\text{env}} \leq 1.5$. Eulerian barriers for $R_E = 5 \text{ Mpc}/h$ are shown: the red curve going through the blue shaded region is the corresponding GR barrier while the blue shaded region shows the modified gravity barriers with the same range of δ_{env} as in the void formation. The green dotted curve shows the spherical collapse approximation [equation (28)] in GR.

lower than in GR. This trend is followed by all curves in Fig. 5, which only differ by their values of s at which $\delta_i - \delta_i^{\text{GR}}$ crosses zero.

To better understand the physics, we shall build up the PLS void assignment scheme step by step, following three steps.

5.2.1 Void barrier (1-barrier)

As discussed in Clampitt, Cai & Li (2013), we fix the environment to be five times the size of the resulting voids. The void formation critical density depends on the environmental density at the corresponding environment. The first crossing distribution is therefore

$$f(S) = \int_0^S ds f(\delta_{\text{env}}, s; R_{\text{env}}) f(S, \delta_v(\delta_{\text{env}})|s, \delta_{\text{env}}), \quad (30)$$

in which $f(\delta_{\text{env}}, s; R_{\text{env}})$ is the first crossing probability of the environmental barrier of Eulerian size R_{env} that corresponds to a void of size S – it is five times the void size. $f(S, \delta_v(\delta_{\text{env}})|s, \delta_{\text{env}})$ is the conditional first crossing probability of the modified void formation criteria density, after having first crossed the environment Eulerian barrier at (s, δ_{env}) . Notice that one does not impose the restriction of not crossing the halo formation barrier here.

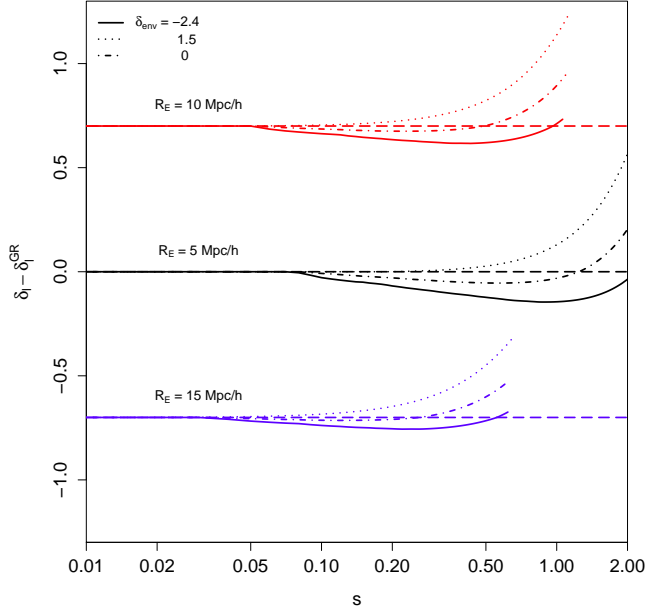


Figure 5. Difference between the modified gravity and GR Eulerian barriers for $R_E = 5, 10, 15 \text{ Mpc/h}$ for three different environmental overdensity. The results for $R_E = 10$ and 15 Mpc/h are shifted by ± 0.7 . We truncate the comparison for $\delta_l < -3$.

5.2.2 Void-in-cloud (2-barrier)

The void-in-cloud effect is straight-forward to implement: the first crossing across an Eulerian barrier corresponding to a halo formation environment of $R_h = 5 \text{ Mpc/h}$ is obtained. The resulting $\delta_{\text{env}}^{\text{halo}}$ will be used to compute the halo formation barrier. We then check that the random walks never cross this halo formation barrier to enforce the void-in-cloud condition.

5.2.3 PLS void assignment

The implementation of the Eulerian-void requires more attention and we will illustrate the idea with the following example. For simplicity assume that we choose a random walk that has not crossed the halo formation barrier. At scale s the random walk has a height δ_l : we would like to get the Eulerian volume V_E at *this point*, and at the moment we neglect previous steps of this walk. In the case of GR, given M and B , one can find the associated Eulerian volume either by looking up the table or by inverting Eq. (28). In the case of modified gravity models, the exact Eulerian volume V_E associated with (s, δ_l) depends on the environmental density, while the size (and density) of the environment depends on V_E , because we have defined void environment to have 5 times the size of the void that forms inside it. We use the following iterative process to break this inter-dependency and obtain V_E :

- (i) Take a guess for the Eulerian size R_E (for example, using the corresponding GR value);
- (ii) We now fix the parameters s, δ_l , and R_E and vary δ_{env} .
- (iii) Set $R_{\text{env}} = 5R_E$ and use the GR numerical table to construct the associated environmental barrier (recall that we assume GR applies in the environment);
- (iv) The first crossing of the environmental barrier associated with R_{env} gives δ_{env} ;
- (v) We now fix three parameters: s, δ_l , and δ_{env} , and vary R_E ;
- (vi) We then search the table of Eulerian barriers in modified gravity models, $B_{V_E}(S|\delta_{\text{env}})$, for an Eulerian volume V'_E that satisfies the three parameters. This gives a new estimate of R'_E ;
- (vii) Repeat step (iii) until we finally arrive at a consistent set of $\{s, \delta_l, R_E, \delta_{\text{env}}\}$. This is deemed to be achieved when R_E changes by less than 1%, or less than 0.5 Mpc/h in absolute value, between two consecutive trials, and we call this the convergence of R_E .

The resulting environmental density contrast δ_{env} is then used to obtain the associated modified gravity void formation barrier. This iterative process is applied at each step of the random walk. We then look for the biggest Eulerian volume that the random walk crossed below the corresponding void formation barrier.

Figure 4 uses a sample random walk to illustrate the idea. We will take the value $s \approx 1$ where the random walk touches the red dashed curve as a demonstration. At this particular value of s , the random walk has a height of around $\delta_l = 0.3$. We now need to find the corresponding Eulerian size for *this* particular value of (s, δ_l) . In the case of GR, this Eulerian size would be 5 Mpc/h . Using this initial guess we obtain an environmental size of $R_{\text{env}} = 25 \text{ Mpc/h}$, whose (GR) Eulerian barrier is shown by the blue dot-dashed curve. The first crossing of this environment barrier is approximately $\delta_{\text{env}} = -0.1$. We then use this δ_{env} value to find a new R_E whose Eulerian barrier will have the value (s, δ_l) . In this particular case the Eulerian barrier for $(R_E, \delta_{\text{env}}) = (5 \text{ Mpc/h}, -0.1)$ lies somewhere in the blue shaded region and hence the new R'_E value will be slightly larger than 5 Mpc/h . One then repeat the above procedure until R_E converges. This iterative process only gives *one* Eulerian size at any particular point in the random walk. To find the largest Eulerian size that would form a void, one needs to examine the whole random walk – which means that the iteration is done at each step of the walk, with a frequency $\Delta s = 0.0025$ for our numerical implementation.

Notice that the implementation of PLS void assignment is computationally very intensive in the case of modified gravity models. For this reason we restrict our algorithm to search only for voids having size bigger than 5 Mpc/h but smaller than 100 Mpc/h . We believe this range is appropriate since smaller voids are difficult to identify and previous study suggests that the signature of modified gravity lies in the bigger voids. On the other hand voids bigger than 100 Mpc/h are extremely rare and this upper limit should not affect our result.

6 RESULTS

Having outlined our methodology, in this section we present the results of the void abundance in our model of modified gravity. In addition to the PLS void assignment scheme we described in the previous section, we also include comparisons with the Lagrangian-void assignment. We will discuss void abundance derived from both correlated and uncorrelated steps in the random walks. In order to avoid sample variance when computing void abundance, we apply the different algorithms on the same set of random walks in both

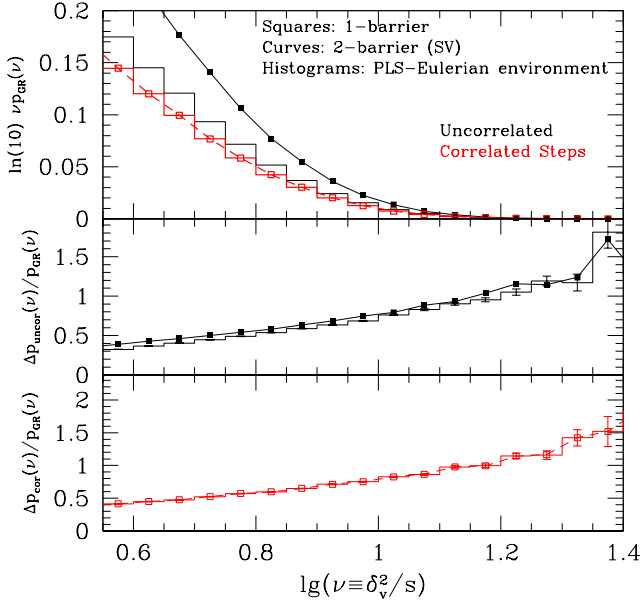


Figure 6. The multiplicity function for various void assigning schemes in the excursion set approach. The top panel shows various results in GR while the middle and the bottom panels show the relative differences in modified gravity models studied in this paper.

the uncorrelated and the correlated cases, which is realised by using the same random k -mode value $\delta(k)$ but different window function $W(kR)$ in

$$\delta(x=0; R) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \delta(k) W(kR), \quad (31)$$

to generate the walks. In the uncorrected case, $W(kR)$ is a top-hat filter in k space – because $\delta(k)$ is a Gaussian random number with different k modes independent of each other, and the k -space top-hat filter does not introduce correlations between them, the values of $\delta(x=0, R)$ are uncorrelated when changing R ; in the correlated case the filter is a top-hat in real space. Note that we can apply the three algorithms (1 barrier, 2 barriers and PLS) to correlated and uncorrelated walks, and therefore there are in total 6 ways to assign voids. We run a total of 40 million random walks to get our results. For the PLS method there are 1476740 (GR) and 2123162 (MG) voids respectively.

Figure 6 shows the first crossing probability of various void assignment schemes. The top panel shows different predictions for GR while the middle and the bottom panels show the differences in modified gravity models. The black histograms/symbols/curves are the results with uncorrelated steps while the red are correlated steps. Note that in both cases we use the top-hat window function to relate the variance s and the smoothing scale in Lagrangian space.

In the case of GR, the three different void assigning schemes have very similar results when correlated steps are used (red symbols), showing our Monte Carlo simulations are consistent with the results in Paranjape, Lam & Sheth (2012b). On the other hand, for the uncorrelated steps, the void abundance in the PLS formalism is significantly lower than the other two. The results for single barrier (solid black squares) and 2-barrier (solid curve) are the same, indicating that at these (relatively large) smoothing scales the void-in-

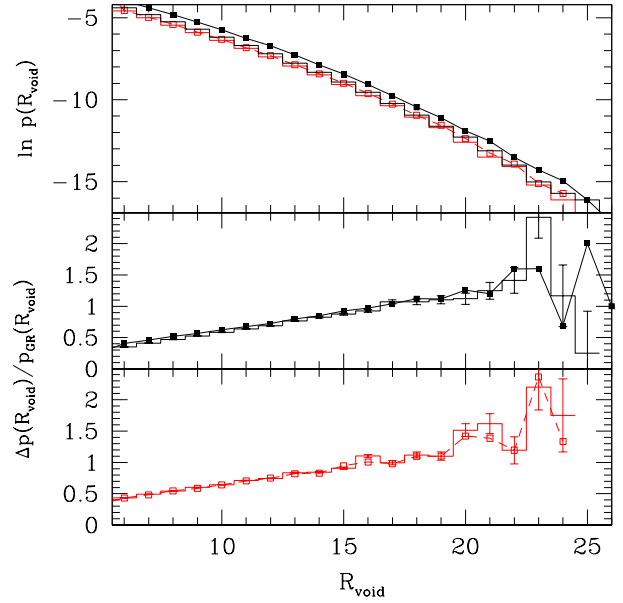


Figure 7. The first crossing probability as a function of the Eulerian void size for various void assigning schemes in the excursion set approach. Same legends are used as in Figure 6. The top panel shows various results in GR while the middle and the bottom panels show modifications in modified gravity models with uncorrelated and correlated steps void assignment schemes respectively.

cloud effect is insignificant (see below). In other words, in the value range of ν (which can be mapped to the void radius, cf. Fig. 7) of interest to us, the expansion of voids are indeed restricted by their environments – the 2-barrier void assignment scheme, which only removes voids-in-cloud that eventually collapses, is over-simplistic and not accurate.

In the middle and the bottom panels we show the modifications in void abundance in modified gravity models using uncorrelated steps and correlated steps in the random walks, respectively. We only show the error bars of one of the schemes (in both cases the histogram for the PLS method) for clarity. In the middle panel the solid squares are the modifications for a single void barrier (hence neglecting the void-in-cloud effect) and the solid curve sharing a very similar result shows the modifications in the 2-barrier case (including void-in-cloud effect). Since it is very unlikely that these large voids are embedded in even larger patches that are going to form halos, the results from these two void assigning methods are nearly the same. Notice that it is not straightforward to directly compare this panel to Fig. 6 in Clampitt, Cai & Li (2013) since we use Eulerian barrier for the environment as well as a different linear power spectrum generated from CAMB (A. Lewis) with the recent Planck cosmology (Planck Collaboration 2014). Nonetheless the trend in the abundance modification is very similar in both studies. The histogram shows the modifications using the PLS algorithm with uncorrelated steps: for very big voids the results are similar to the other two void algorithms, however there are differences when going to relatively small voids. The modification in void abundance using the PLS algorithm is slightly less than the other algorithms in small voids.

The bottom panel shows the modification of void abundance in modified gravity models for correlated steps. In this case we em-

ploy the top-hat filter in real space and generate a series of random walks. The open squares, the dashed curve, and the histogram represent results from the 1-barrier, 2-barrier, and PLS void algorithm respectively. All of them show very similar results, suggesting that when using correlated steps, one would derive the void abundance using the much simpler 1-barrier or 2-barrier algorithms. We also checked that while there are some quantitative differences, the results from uncorrelated steps random walk with 1-barrier or 2-barrier algorithm agree with the more sophisticated correlated steps with PLS algorithm.

In summary, while there are noticeable differences in the void abundances from the different void assignment schemes, the relative difference between our chameleon model and GR only changes slightly. This is somewhat surprising, but it may simply suggest that void abundance in chameleon models changes with different voids assignment in a similar manner as in GR.

Figure 7 shows the first crossing distribution plotted as a function of the Eulerian sizes of voids being formed. This figure uses the same set of random walks, but since s and R are not linearly related the walks are organised into different bins. We use the same legends as in Figure 6. One noticeable difference when comparing the middle and the bottom panels with those of Figure 6 is stronger modifications at very large voids: although very rare, the abundance of large voids (larger than 20 Mpc/h) is a very sensitive probe of modified gravity models. Notice that the model parameters used in this work are mild such that the fifth force helps to enhance the halo mass function by $\lesssim 15\%$ across the whole mass range at $z = 0$ (Li & Efstathiou 2012); in contrast, for the void radii studied in this work, we can see that the fifth force can enhance the void abundance by over 50% (and over 100% for $R_{\text{void}} > 15$ Mpc/h).

7 SUMMARY AND DISCUSSION

In this paper we considered the void abundance in the chameleon-type modified gravity models – we have extended previous work in Clampitt, Cai & Li (2013) and Paranjape, Lam & Sheth (2012b) to compute the void volume function in modified gravity models, using the recently developed Eulerian void assignment (PLS) scheme within the excursion set approach and to compare its result to other void assignment schemes. This Eulerian void method includes the effect of the surrounding on the growth of void – it would reduce the size of the void or even disqualify it as a void (if the surrounding collapses). A brief description of this PLS void method is included in section 5.1.

In order to implement this PLS void assignment method in modified gravity models, we solved the spherical collapse equation of motion for various environmental density contrasts, to calculate the critical density contrasts for halo and void formation. We performed validation test on the numerical solver to make sure the barriers being used in the excursion set approach are correct. We then applied these barriers to evaluate the first crossing probability using Monte Carlo simulations: we used both correlated and uncorrelated steps random walks on various void assigning schemes. Correlated and uncorrelated steps random walks are associated with the window function kernel used to define the Lagrangian patches: the former corresponds to top-hat or Gaussian window functions while the latter to a sharp- k filter.

We then discussed the implementation of the excursion set approach for void volume function in modified gravity models in section 5.2. The 1-barrier and 2-barrier cases are relatively straightforward, while we introduce an iterative method in the PLS for-

malism to obtain a consistent description of the parameter set $\{s, \delta_t, R_E, \delta_{\text{env}}\}$ – it is important since the evolution of structure depends on the strength of the fifth force, and hence the environmental density contrast. Notice that these are implemented using the Eulerian picture.

The results of our calculation are summarised in Figs. 6 and 7. Our results are consistent with those in Clampitt, Cai & Li (2013): the abundances of voids are very sensitive to modified gravity. One unexpected finding is the modifications in the void volume function due to modified gravity models using the 1-barrier and 2-barrier methods with uncorrelated steps match those using the PLS method with correlated steps very well. We believe this would be a coincidence – the fact that the bases of comparison (i.e., the GR predictions, see the top panel) are very different supports it¹. On the other hand, the results of the relatively straightforward 1-barrier with correlated steps agree with those of the PLS algorithm, for both the GR results as well as the modifications due to modified gravity models. Hence when dealing with correlated steps, it is sufficient to compute the void abundance using the 1-barrier method – either with Monte Carlo simulations as we did in this work or with one of the approximations developed for computing first crossing probability.

The enhancement of void abundances due to modified gravity model shows different characteristic than that of halo abundances: the chameleon screening is less efficient for these under-dense regions and hence the abundances of voids (in particular large ones) are very sensitive to modified gravity. Furthermore, there is detriment in the abundance of low mass halos due to mass conservation while the void abundance is always enhanced (down to the scale we investigate). By combining the abundances of halos and voids it is possible to break the degeneracy between modified gravity models and other cosmological parameters or models, such as σ_8 and massive neutrinos, a possibility which merits further investigations in the future.

While void abundance is a sensitive probe of modified gravity models, there are several complications needing to be addressed before it can be used to put constraints on these models. First, real void catalogues are constructed using biased tracers in galaxy surveys, while our study assumes that voids are found from the dark matter field. This may introduce biases – since modified gravity modifies halo abundance (hence galaxies) as well, such biases may be different for GR and modified gravity models. Second, the predictions from the excursion set approach do not agree with the void catalogues generated using dark matter particles simulations, even for the Λ CDM paradigm (see, for example, Jennings, Li & Hu 2013; Achitouv, Neyrinck & Paranjape 2013). Different methods are proposed to circumvent this discrepancy but it is important to check for consistency in both GR and modified gravity models if any of these methods was used in the comparison analysis. Third, the increase of void abundance in modified gravity models aggravates the problem of having a void volume fraction larger than unity. Notice that the excursion set approach only considers structure formation at each position *independently* – while this may be reasonable for collapsing objects (halos), objects (voids) having their comoving sizes expanded may have a larger chance to merge with each other, invalidating the assumption of the excursion set approach – this problem will be worse in the presence of the fifth

¹ Another possibility is that the application of the PLS void assignment method with correlated steps introduces the same differences to the simpler 1-barrier or 2-barrier results, and when taking the relative change this gets cancelled out to a large extent.

force, which helps inflate the voids faster. Fourth, given that the fifth force is less efficiently suppressed in voids, the evolution and properties of galaxies there can also be different from in GR, adding more subtlety to the interpretation of observational data and constraints using them.

Finally, as pointed out in Clampitt, Cai & Li (2013), since voids sitting in over-dense environments show stronger signatures of modified gravity models (at least for the chameleon-type theories), one would possibly increase the signal-to-noise by using the conditional void volume function, or by constructing a weighted void volume function. The latter method is now being investigated with both weighted halo mass function as well as weighted void volume function.

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REFERENCES

- Achitouv I., Neyrinck M., Paranjape A., 2013, arXiv: 1309.3799
- Planck Collaboration, Ade P. A. R., *et al.*, 2014, A & A, *in press*
- Bernardeau F., 1994, ApJ, 427, 51
- Bond J. R., Cole S., Efstathiou G., Kaiser N., 1991, ApJ, 379, 440
- Bos E. G. P., van de Weygaert R., Dolag K., Pettorino V., 2012, MNRAS, 426, 440
- Cai Y.-C., Li B., Cole S., Frenk C. S., Neyrinck M., 2014, MNRAS, 439, 2978
- Cai Y.-C., Neyrinck M. C., Szapudi I., Cole S., Frenk C. S., 2014, ApJ, 786, 110
- Clampitt J., Cai Y.-C., Li B., 2013, MNRAS, 431, 749
- Clampitt J., & Jain, B. 2014, arXiv:1404.1834
- Clifton, T., Ferreira, P. G., Padilla, A., Skordis, C., 2012, PhR, 513, 1
- Corasaniti P. S., Achitouv I., 2011, PRD, 023009
- Davis A. C., Lim E. A., Sakstein J., Shaw D. J., 2012, PRD, 85, 123006
- Granett B. R., Neyrinck M. C., Szapudi I., 2008, ApJL, 683, L99
- Hamaus, N., Wandelt, B. D., Sutter, P. M., Lavaux, G., & Warren, M. S. 2014, Phys. Rev. Lett., 112, 041304
- Hotchkiss S., Nadathur S., Gottlöber S., Iliev I. T., Knebe A., Watson W. A., Yepes G., 2014, ArXiv e-prints
- Hoyle, F., & Vogeley, M. S. 2002, ApJ, 566, 641
- Hoyle, F., Vogeley, M. S., & Pan, D. 2012, MNRAS, 426, 3041
- Hu W., Sawicki I., 2007, PRD, 76, 064004
- Ilić S., Langer M., Douspis M., 2013, Astron. Astrophys., 556, A51
- Jain B., Khoury J., 2010, Annals of Physics, 325, 1479
- Jennings E., Li Y., Hu W., 2013, 434, 2167
- Khoury J., Weltman A., 2004, PRD, 69, 044026
- Lam T. Y., Li B., 2012, MNRAS, 426, 3260
- Lam T. Y., Sheth R. K., 2008a, MNRAS, 386, 407
- Lam T. Y., Sheth R. K., 2008b, MNRAS, 389, 1249
- Lam T. Y., Sheth R. K., 2009, MNRAS, 398, 2143
- Lam T. Y., Sheth R. K., Desjacques V., 2009, MNRAS, 399, 1482
- Lavaux, G., & Wandelt, B. D. 2012, ApJ, 754, 109
- Lewis A., <http://camb.info/>
- Li B., 2011, MNRAS, 411, 2615
- Li B., Barrow J. D., 2007, PRD, 75, 084010
- Li B., Barrow J. D., 2011, PRD, 83, 024007
- Li B., Efstathiou G., 2012, MNRAS, 421, 1431
- Li B., Zhao G.-B., Koyama K., 2012, MNRAS, 421, 3481
- Li B., Zhao H., 2009, PRD, 80, 044027
- Li B., Zhao H., 2010, PRD, 81, 104047
- Li B., Zhao G., Teyssier R., Koyama K., 2012, JCAP, 01, 051
- Li B., Lam T. Y., 2012, MNRAS, 425, 730
- Lombriser L., Koyama K., Zhao G., Li B., 2011, PRD, 85, 124054
- Ma C., Maggiore M., Riotto A., Zhang J., 2011, MNRAS, 411, 2644
- Maggiore M., Riotto A., 2010, ApJ, 711, 907
- Maggiore M., Riotto A., 2010, ApJ, 717, 515
- Melchior, P., Sutter, P. M., Sheldon, E. S., Krause, E., & Wandelt, B. D. 2013, arXiv:1309.2045
- Mo H. J., White S. D. M., 1996, MNRAS, 282, 347
- Musso M., Sheth R. K., 2012, MNRAS, 423, L102
- Oyaizu H., 2008, PRD, 78, 123523
- Oyaizu H., Lima M., Hu W., 2008, PRD, 78, 123524
- Pan, D. C., Vogeley, M. S., Hoyle, F., Choi, Y.-Y., & Park, C. 2012, MNRAS, 421, 926
- Paranjape A., Sheth R. K., 2012, MNRAS, 426, 2789
- Paranjape A., Lam T. Y., Sheth R. K., 2012, MNRAS, 420, 1429
- Paranjape A., Lam T. Y., Sheth R. K., 2012, MNRAS, 420, 1648
- Paranjape A., Sheth R. K., Desjacques V., 2013, MNRAS, 431, 1503
- Planck Collaboration Ade P. A. R., Aghanim N., Armitage-Caplan C., Arnaud M., Ashdown M., Atrio-Barandela F., Aumont J., Baccigalupi C., Banday A. J., *et al.* 2013, ArXiv e-prints
- Schmidt F., Lima M., Oyaizu H., Hu W., 2009, PRD, 79, 083518
- Sheth R. K., 1998, MNRAS, 300, 1057
- Sheth R. K., Tormen G., 1999, MNRAS, 308, 119
- Sheth R. K., Tormen G., 2002, MNRAS, 329, 61
- Sheth R. K., van de Weygaert R., 2004, MNRAS, 350, 517
- Sutter, P. M., Lavaux, G., Wandelt, B. D., & Weinberg, D. H. 2012, ApJ, 761, 187
- Sutter P. M., Carlesi E., Wandelt B. D., Knebe A., 2014, ArXiv e-prints
- Sutter, P. M., Pisani, A., Wandelt, B. D., & Weinberg, D. H. 2014, arXiv:1404.5618
- Zentner A. R., 2007, IJMPD, 16, 763
- Zhang J., Hui L., 206, ApJ, 641, 641
- Zhao G., Li B., Koyama K., 2011, PRD, 83, 044007